لجنة الميكانيك
تقدم لكم..

المكتبة التخصصية

لجنة الميكانيك
Polytechnic

Mech.MuslimEngineer.Net

FB.com/Groups/Mid.Group

0789434018

MechFet

Chapter 9
Static-Force Analysis

9.1 INTRODUCTION

A machine is a device that performs work and, as such, transmits energy by means of mechanical force from a power source to a driven load. It is necessary in the design of machine mechanisms to know the manner in which forces are transmitted from the input to the output, so that the components of the machine can be properly sized to withstand the stresses that are developed. If the members are not designed to be strong enough, then failure will occur during machine operation; if, on the other hand, the machine is overdesigned to have much more strength than required, then the machine may not be competitive with others in terms of cost, weight, size, power requirements, or other criteria.

Various assumptions must be made during the course of a force analysis reflecting the specific characteristics of the particular system being investigated. Whenever possible, these assumptions should be verified as the design proceeds. A major assumption concerns dynamic or inertia forces. All machines have mass, and if parts of a machine are accelerating, there will be inertia forces associated with this motion. If the magnitudes of these inertia forces are small relative to externally applied loads, then they can be neglected in the force analysis. Such an analysis is referred to as static-force analysis and is the topic of this chapter. For example, during normal operation of a front-end loader, such as that shown later in the

chapter in Figure 9.13A, the bucket load and static weight loads may far exceed any dynamic loads due to accelerating masses, and a static-force analysis would be justified. An analysis that includes inertia effects is called a dynamic-force analysis and will be discussed in the next chapter. An example of an application where a dynamic-force analysis would be required is in the design of an automatic sewing machine, where, due to high operating speeds, the inertia forces may be greater than the external loads on the machine.

Another assumption deals with the rigidity of the machine components. No material is truly rigid, and all materials will experience significant deformation if the forces, either external or inertial in nature, are great enough. It will be assumed in this chapter and the next that deformations are so small as to be negligible and, therefore, the members will be treated as though they are rigid. The subject of mechanical vibrations, which is beyond the scope of this book, considers the flexibility of machine components and the resulting effects on machine behavior.

A third major assumption that is often made is that friction effects are negligible. Friction is inherent in all devices, and its degree is dependent upon many factors, including types of bearings, lubrication, loads, environmental conditions, and so on. Friction will be neglected in the first few sections of this chapter, with an introduction to the subject presented in Section 9.5.

In addition to assumptions of the types discussed above, other assumptions may be necessary, and some of these will be addressed at various points throughout the chapter.

The first part of this chapter is a review of general force analysis principles and will also establish some of the convention and terminology to be used in succeeding sections. The remainder of the chapter will then present both graphical and analytical methods for static-force analysis of machines.

9.2 BASIC PRINCIPLES OF FORCE ANALYSIS

9.2.1 Force and Torque

A force is a vector quantity; it has a magnitude, a direction or line of action, and a sense. Figure 9.1 shows a force vector F, which can be expressed in terms of

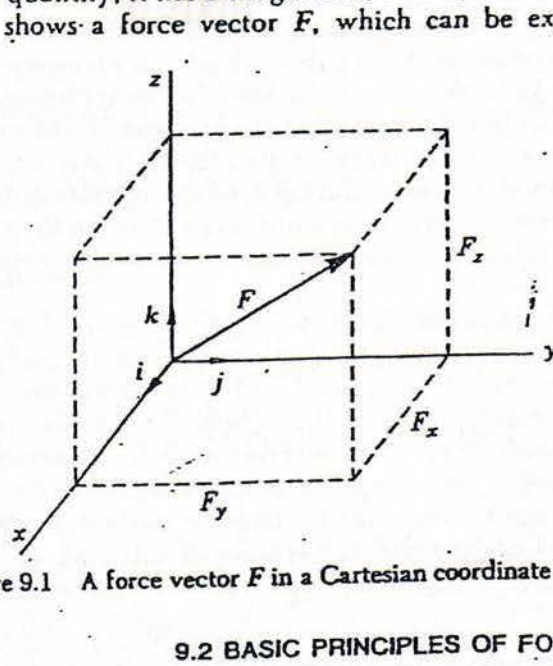


Figure 9.1 A force vector F in a Cartesian coordinate system.

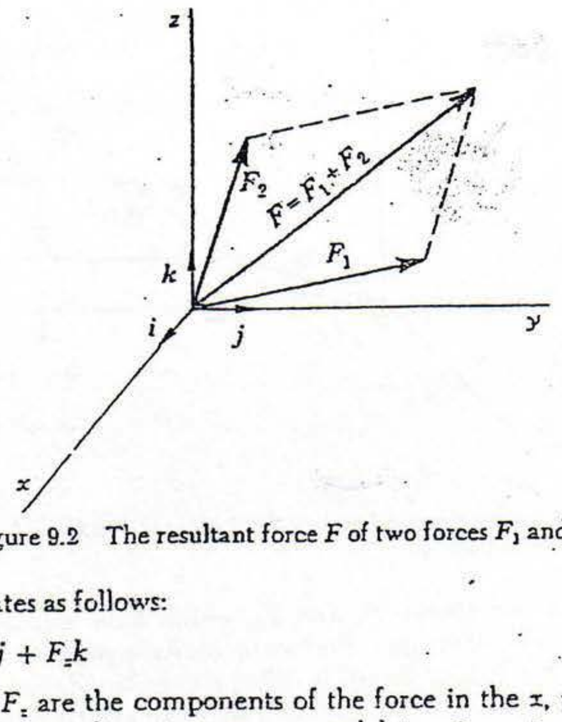


Figure 9.2 The resultant force F of two forces F_1 and F_2.

Cartesian coordinates as follows:

$$F = F_x i + F_y j + F_z k \tag{9.1}$$

where F_x, F_y, and F_z are the components of the force in the x, y, and z directions, respectively, with these directions represented by unit vectors i, j, and k. The resultant force F of two forces F_1 and F_2 is the vector sum of those forces. This is expressed graphically in Figure 9.2 and mathematically as follows:

$$F = F_1 + F_2 = (F_{1x} + F_{2x})i + (F_{1y} + F_{2y})j + (F_{1z} + F_{2z})k \tag{9.2}$$

where F_{1x} is the x component of force F_1, and so on.

A torque or moment T is defined as the moment of a force about a point, and it also is a vector quantity. Using the vector cross product notation, we have

$$\boxed{T = R \times F} \tag{9.3}$$

where R is a position vector directed from the point about which the moment is taken to any point on the line of action of force F. See Figure 9.3. The magnitude of T is

$$T = RF|\sin\theta|$$

where θ is the angle between vectors R and F, and R and F are the magnitudes of the vectors. The direction of T is perpendicular to the plane containing R and F and the sense is given by the right-hand rule. Alternatively, in determinant form,

$$T = \begin{vmatrix} i & j & k \\ R_x & R_y & R_z \\ F_x & F_y & F_z \end{vmatrix} \tag{9.4}$$

$$= (R_yF_z - R_zF_y)i + (R_zF_x - R_xF_z)j + (R_xF_y - R_yF_x)k$$

An infinite number of combinations of a force vector F and a moment arm vector R exist that will produce the same moment T; that is, different values of vectors R and F can lead to the same cross product as given by Eq. 9.3. The resultant of two or more moments is the vector sum of the moments.

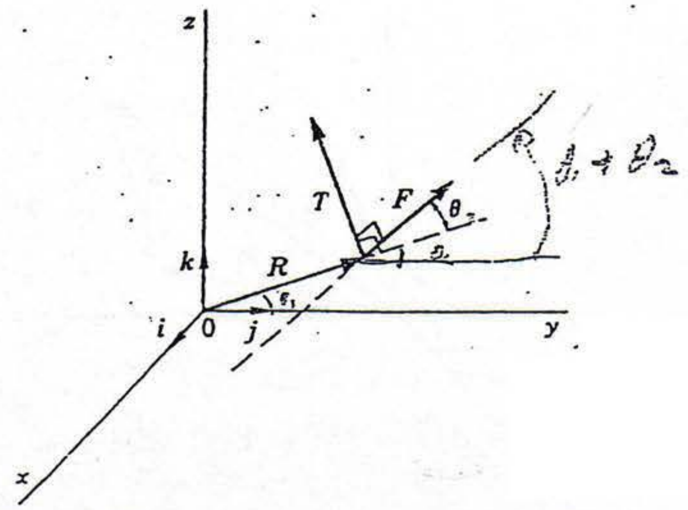


Figure 9.3 Torque T is the moment of force F about point 0. Vector R locates the line of action of the force relative to point 0.

Figure 9.4 shows two forces, F_1 and F_2, which have equal magnitudes but different lines of action. Furthermore, the two forces have parallel directions and are of opposite sense. Such a pair of forces is called a couple. The resultant force of a couple is zero. However, the resultant moment about an arbitrary point is not zero.

For example, if moments about the origin in Figure 9.4 are summed, the resultant moment T is

$$T = R_1 \times F_1 + R_2 \times F_2$$

But $F_1 = -F_2$, and, therefore,

$$T = R_1 \times (-F_2) + R_2 \times F_2 = (R_2 - R_1) \times F_2 = R \times F_2 \qquad (9.5)$$

where $R = R_2 - R_1$ is a vector from any point on the line of action of F_1 to any point on the line of action of F_2. The direction of the torque is perpendicular to the plane of

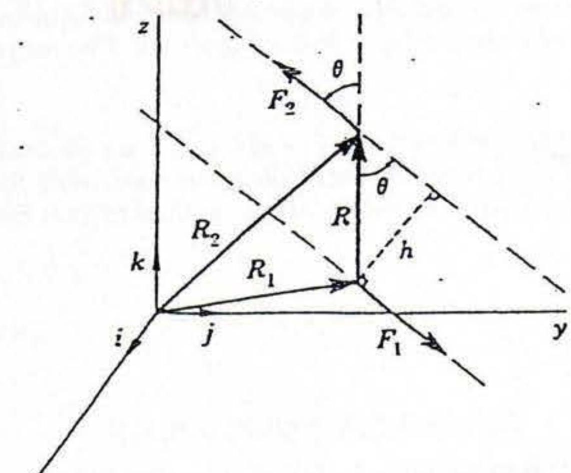


Figure 9.4 Forces F_1 and F_2 form a couple, which has zero resultant force but a nonzero resultant moment.

the couple and the magnitude is given by

$$T = RF_2|\sin\theta| = hF_2 \tag{9.6}$$

where $h = R|\sin\theta|$ is the perpendicular distance between the lines of action. It can be seen that the resultant moment of a couple, Eq. 9.5, is independent of the point about which moments are taken. Conversely, the moment of a couple about a particular point is independent of the position of the couple relative to the point. For these reasons, a couple is sometimes referred to as a pure moment or pure torque. As will be seen, the concept of a couple is very useful in force analysis applications.

9.2.2 Free-Body Diagrams

Engineering experience has demonstrated the importance and usefulness of free-body diagrams in force analysis. A free-body diagram is a sketch or drawing of part or all of a system, isolated in order to determine the nature of forces acting on that body. Sometimes a free-body diagram may take the form of a mental picture; however, actual sketches are strongly recommended, especially for complex mechanical systems.

Generally, the first, and one of the most important, steps in a successful force analysis is the identification of the free bodies to be used. Figures 9.5B through 9.5E show examples of various free bodies that might be considered in the analysis of the four-bar linkage shown in Figure 9.5A. In Figure 9.5B, the free body consists of the three moving members isolated from the frame; here, the forces acting on the free body include a driving force or torque, external loads, and the forces transmitted

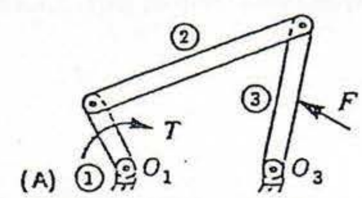


Figure 9.5(A) A four-bar linkage.

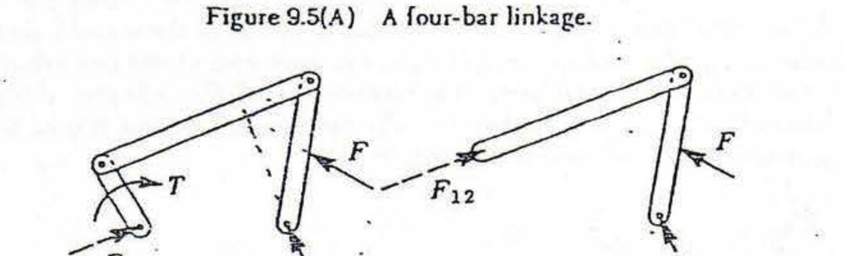


Figure 9.5(B) Free-body diagram of the three moving links. (C) Free-body diagram of two connected links.

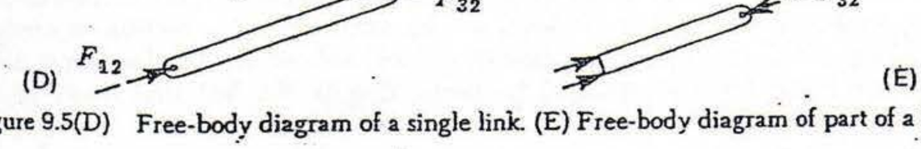


Figure 9.5(D) Free-body diagram of a single link. (E) Free-body diagram of part of a link.

from the frame at bearings O_1 and O_3. The force convention is defined as follows: F_{ij} represents the force exerted by member i on member j. This convention will be used throughout the text. Figure 9.5C is a free-body diagram of two links, which are acted upon by the forces transmitted from adjoining links as well as other applied loads. Probably the most commonly used form of a free-body diagram is that of a single link. See Figure 9.5D. Most force analyses can be accomplished by examining each of the individual members that make up the system. Such an approach leads to the determination of all of the bearing forces between members as well as the required input force or torque for a given output load or set of loads. For investigating internal forces or stresses in members, free bodies consisting of portions of members, as in Figure 9.5E, are useful.

9.2.3 Static Equilibrium

For a free body in static equilibrium, the vector sum of all forces acting on the body must be zero and the vector sum of all moments about any arbitrary point must also be zero. These conditions can be expressed mathematically as follows:

$$\sum F = 0 \tag{9.7A}$$

$$\sum T = 0 \tag{9.7B}$$

Since each of these vector equations represents three scalar equations, there are a total of six independent scalar conditions that must be satisfied for the general case of equilibrium under three-dimensional loading

There are many situations where the loading is essentially planar, in which case, forces can be described by two-dimensional vectors. If the xy plane designates the plane of loading, then the applicable form of Eqs. 9.7A and 9.7B is

$$\sum F_x = 0 \tag{9.8A}$$

$$\sum F_y = 0 \tag{9.8B}$$

$$\sum T_z = 0 \tag{9.8C}$$

Eqs. 9.8A to 9.8C are three scalar equations that state that, for the case of two-dimensional xy loading, the summations of forces in the x and y directions must individually equal zero and the summation of moments about any arbitrary point in the plane must also equal zero. The remainder of this chapter deals with two-dimensional force analysis. A common example of three-dimensional forces is gear forces, which were discussed in Chapter 7.

9.2.4 Superposition

The principle of superposition of forces is an extremely useful concept, particularly in graphical force analysis. Basically, the principle states that, for linear systems, the net effect of multiple loads on a system is equal to the superposition (i.e., vector summation) of the effects of the individual loads considered one at a time. Physically, linearity refers to a direct proportionality between input force and output force. Its mathematical characteristics will be discussed in the section on analytical force analysis. Generally, in the absence of Coulomb or dry friction, most mechanisms are linear for force analysis purposes, despite the fact that many of these

mechanisms exhibit very nonlinear motions. Examples and further discussion in later sections will demonstrate the application of this principle.

9.3 GRAPHICAL FORCE ANALYSIS

Graphical force analysis employs scaled free-body diagrams and vector graphics in the determination of unknown machine forces. The graphical approach is best suited for planar force systems. Since forces are normally not constant during machine motion, analyses may be required for a number of mechanism positions; however, in many cases, critical maximum-force positions can be identified and graphical analyses performed for these positions only. An important advantage of the graphical approach is that it provides useful insight as to the nature of the forces in the physical system.

This approach suffers from disadvantages related to accuracy and time. As is true of any graphical procedure, the results are susceptible to drawing and measurement errors. Further, a great amount of graphics time and effort can be expended in the iterative design of a machine mechanism for which fairly thorough knowledge of force-time relationships is required. In recent years, the physical insight of the graphics approach and the speed and accuracy inherent in the computer-based analytical approach have been brought together through computer graphics systems, which have proven to be very effective engineering design tools.

There are a few special types of member loadings that are repeatedly encountered in the force analysis of mechanisms. These include a member subjected to two forces, a member subjected to three forces, and a member subjected to two forces and a couple. These special cases will be considered in the following paragraphs, before proceeding to the graphical analysis of complete mechanisms.

9.3.1 Analysis of a Two-Force Member

A member subjected to two forces is in equilibrium if and only if the two forces (1) have the same magnitude, (2) act along the same line, and (3) are opposite in sense. Figure 9.6A shows a free-body diagram of a member acted upon by forces F_1 and F_2, where the points of application of these forces are points A and B. For equilibrium, the directions of F_1 and F_2 must be along line AB and F_1 must equal $(-F_2)$. A graphical vector addition of forces F_1 and F_2 is shown in Figure 9.6B, and, obviously, the resultant net force on the member is zero when $F_1 = -F_2$. The resultant moment about any point will also be zero, as can be seen from inspection or by application of Eq. 9.5.

Thus, if the load application points for a two-force member are known, the line of action of the forces is defined, and if the magnitude and sense of one of the forces are known, then the other force can immediately be determined. Such a member will either be in tension or compression.

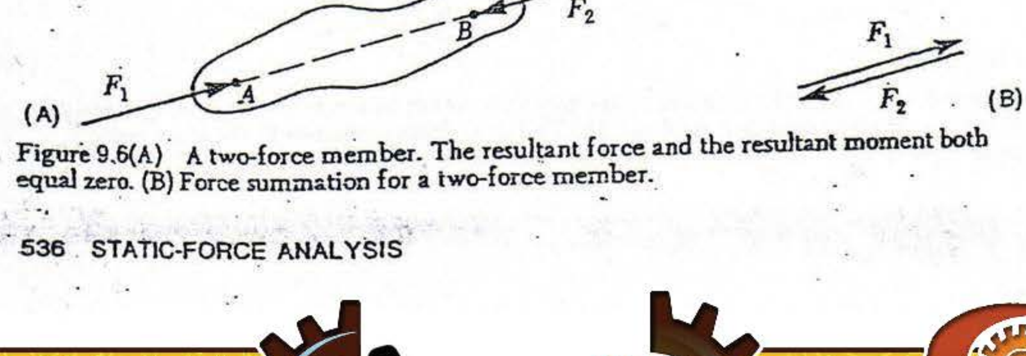


Figure 9.6(A) A two-force member. The resultant force and the resultant moment both equal zero. (B) Force summation for a two-force member.

9.3.2 Analysis of a Three-Force Member

A member subjected to three forces is in equilibrium if and only if (1) the resultant of the three forces is zero, and (2) the lines of action of the forces all intersect at the same point. The first condition guarantees equilibrium of forces, while the second condition guarantees equilibrium of moments. The second condition can be understood by considering the case when it is not satisfied. See Figure 9.7A. If moments are summed about point P, the intersection of forces F_1 and F_2, then the moments of these forces will be zero, but F_3 will produce a nonzero moment, resulting in a nonzero net moment on the member. On the other hand, if the line of action of force F_3 also passes through point P (Figure 9.7B), the net moment will be zero. This common point of intersection of the three forces is called the point of concurrency.

A typical situation encountered is that when one of the forces, F_1, is known completely, magnitude and direction, a second force, F_2, has known direction but unknown magnitude, and force F_3 has unknown magnitude and direction. The graphical solution of this case is depicted in Figures 9.8A through 9.8C. First, the free-body diagram is drawn to a convenient scale and the points of application of the three forces are identified. These are points A, B, and C. Next, the known force F_1 is drawn on the diagram with the proper direction and a suitable magnitude scale. The direction of force F_2 is then drawn, and the intersection of this line with an extension of the line of action of force F_1 is the concurrency point P. For equilibrium, the line of action of force F_3 must pass through points C and P and is therefore as shown in Figure 9.8A.

The force equilibrium condition states that

$$F_1 + F_2 + F_3 = 0$$

Since the directions of all three forces are now known and the magnitude of F_1 was given, this equation can be solved for the remaining two magnitudes. A graphical solution follows from the fact that the three forces must form a closed vector loop, called a force polygon. The procedure is shown in Figure 9.8B. Vector F_1 is redrawn. From the head of this vector, a line is drawn in the direction of force F_2, and from the tail, a line is drawn parallel to F_3. The intersection of these lines closes the vector loop and determines the magnitudes of forces F_2 and F_3. Note that the same solution is obtained if, instead, a line parallel to F_3 is drawn from the head of F_1, and a line parallel to F_2 is drawn from the tail of F_1. See Figure 9.8C. This is so because vector

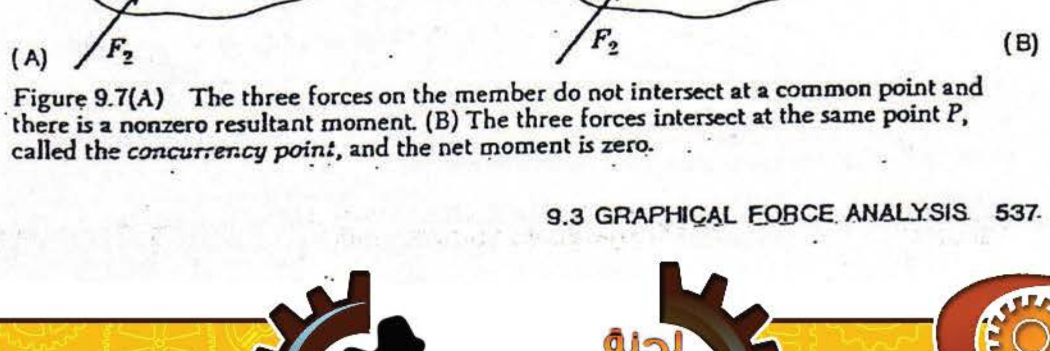


Figure 9.7(A) The three forces on the member do not intersect at a common point and there is a nonzero resultant moment. (B) The three forces intersect at the same point P, called the *concurrency point*, and the net moment is zero.

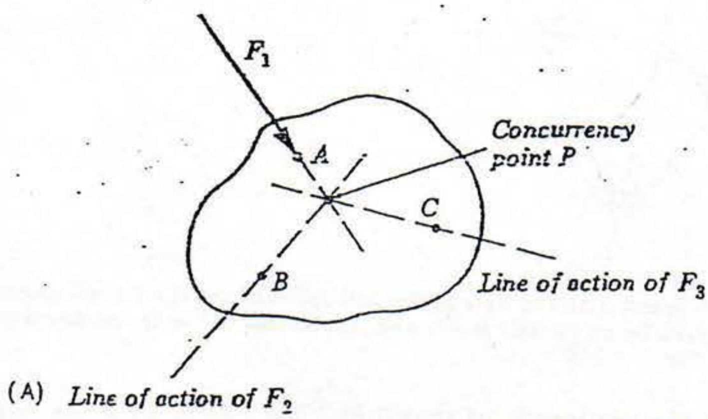


Figure 9.8(A) Graphical force analysis of a three-force member.

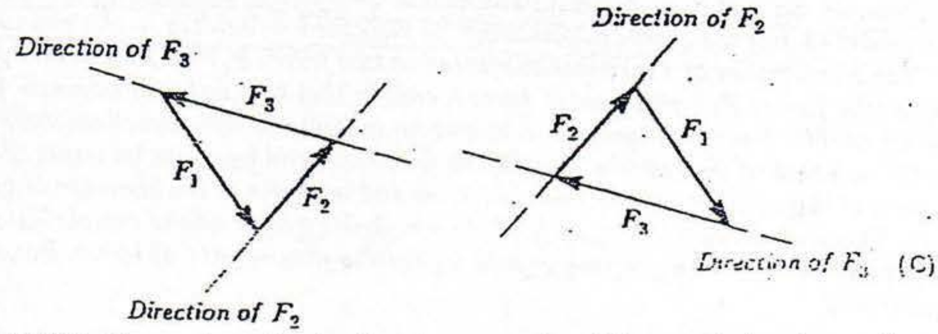


Figure 9.8(B) Force polygon for the three-force member. (C) An equivalent force polygon for the three-force member.

addition is commutative, and, therefore, both force polygons are equivalent to the vector equation above.

It is important to remember that, by the definition of vector addition, the force polygon corresponding to the general force equation

$$\sum F = 0$$

will have adjacent vectors connected head to tail. This principle is used in identifying the sense of forces F_2 and F_3 in Figures 9.8B and 9.8C. Also, if the lines of action of F_1 and F_2 are parallel, then the point of concurrency is at infinity, and the third force F_3 must be parallel to the other two. In this case, the force polygon collapses to a straight line.

9.3.3 Analysis of a Member with Two Forces and a Couple

In performing force analyses, it is imperative that we know the nature of forces that drive the system or that act as loads on the system. Only by knowing where these forces act and how they act can we proceed with a complete force analysis of the system.

This point can be illustrated by considering two ways by which the input crank of a four-bar linkage can be driven. See Figures 9.9A and 9.9B. In Figure 9.9A, the bell crank is driven by a hydraulic cylinder attached at the point shown. In this case, the crank is a three-force member and the analysis proceeds according to the preceding section. If, on the other hand, the crank is driven by a shaft with a direct

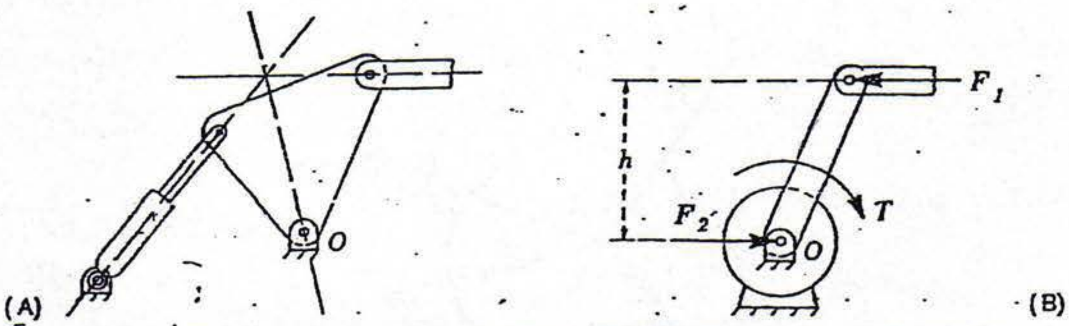


Figure 9.9(A) The crank is driven by a piston and cylinder and is a three-force member. (B) The crank is driven by an electric motor and is subjected to two forces and a pure torque.

connection to an electric motor, as shown in Figure 9.9B, then the applied shaft torque takes the form of a couple, and the crank is subjected to two forces plus this input couple. Both of these drive systems can be designed to produce the same torque about pivot O, but the forces experienced by the crank will differ in the two cases.

For equilibrium of a member subjected to two forces $\mathbf{F}_1$ *and* $\mathbf{F}_2$ *plus an applied couple, the forces* $\mathbf{F}_1$ *and* $\mathbf{F}_2$ *must form a couple that is equal and opposite to the applied couple.* Hence, if force $\mathbf{F}_1$ is known in magnitude and direction, then force $\mathbf{F}_2$ will be equal in magnitude, parallel in direction, and opposite in sense, and the moment of the applied couple must be equal and opposite to the moment of couple $\mathbf{F}_1, \mathbf{F}_2$. This is illustrated in Figure 9.9B, in which the magnitude of couple T is equal to the product $hF_1 = hF_2$, where F_1 and F_2 are the magnitudes of forces $\mathbf{F}_1$ and $\mathbf{F}_2$, respectively.

9.3.4 Graphical Force Analysis of the Slider Crank Mechanism

The slider crank mechanism finds extensive application in reciprocating compressors, piston engines, presses, toggle devices, and other machines where force characteristics are important. The force analysis of this mechanism employs most of the principles described in previous sections, as demonstrated by the following example.

Example Problem 9.1

Static-force analysis of a slider crank mechanism is discussed. Consider the slider crank linkage shown in Figure 9.10A, representing a compressor, which is operating

$OB = 30\ mm$
$BC = 70\ mm$
$\phi = 45°$

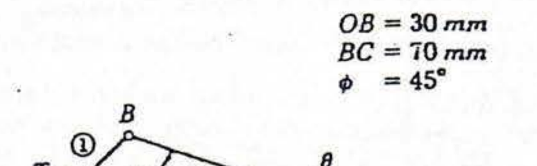


Figure 9.10(A) Graphical force analysis of a slider crank mechanism, which is acted on by piston force P and crank torque T.

at so low a speed that inertia effects are negligible. It is also assumed that gravity forces are small compared with other forces and that all forces lie in the same plane. The dimensions are $OB = 30$ mm and $BC = 70$ mm. We wish to find the required crankshaft torque T and the bearing forces for a total gas pressure force $P = 40$ N at the instant when the crank angle $\phi = 45°$.

SOLUTION

The graphical analysis is shown in Figure 9.10B. First, consider connecting rod 2. In the absence of gravity and inertia forces, this link is acted on by two forces only, at pins B and C. These pins are assumed to be frictionless and, therefore, transmit no torque. Thus, link 2 is a two-force member loaded at each end as shown. The forces F_{12} and F_{32} lie along the link, producing zero net moment, and must be equal and opposite for equilibrium of the link. At this point, the magnitude and sense of these forces are unknown.

Next, examine piston 3, which is a three-force member. The pressure force P is completely known and is assumed to act through the center of the piston (i.e., the pressure distribution on the piston face is assumed to be symmetric). From Newton's third law, which states that for every action there is an equal and opposite reaction, it follows that $F_{23} = -F_{32}$, and the direction of F_{23} is therefore known. In the absence of friction, the force of the cylinder on the piston, F_{03}, is perpendicular to the cylinder wall, and it also must pass through the concurrency point, which is the piston pin C. Now, knowing the force directions, we can construct the force polygon for member 3 (Figure 9.10B). Scaling from this diagram, the contact force between the cylinder and piston is $F_{03} = 12.7$ N, acting upward and the magnitude of the bearing force at C is $F_{23} = F_{32} = 42.0$ N. This is also the bearing force at crankpin B, because $F_{12} = -F_{32}$. Further, the force directions for the connecting rod shown in the figure are correct, and the link is in compression.

Finally, crank 1 is subjected to two forces and a couple T (the shaft torque T is assumed to be a couple). The force at B is $F_{21} = -F_{12}$ and is now known. For force equilibrium, $F_{01} = -F_{21}$, as shown on the free-body diagram of link 1. However, these forces are not colinear, and for equilibrium, the moment of this couple must be

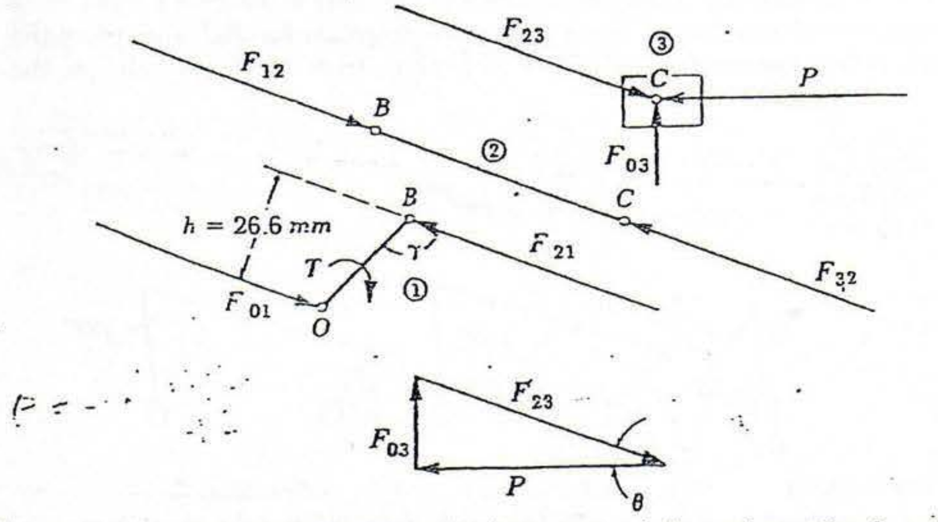


Figure 9.10(B) Static force balances for the three moving links, each considered as a free body.

balanced by torque T. Thus, the required torque is clockwise and has magnitude

$$T = F_{21}h = (42.0\ \text{N})\,(26.6\ \text{mm}) = 1120\ \text{N}\cdot\text{mm} = 1.12\ \text{N}\cdot\text{m}$$

It should be emphasized that this is the torque required for static equilibrium in the position shown in Figure 9.10A. If torque information is needed for a complete compression cycle, then the analysis must be repeated at other crank positions throughout the cycle. In general, the torque will vary with position.

9.3.5 Graphical Force Analysis of the Four-Bar Linkage

The force analysis of the four-bar linkage proceeds in much the same manner as that of the slider crank mechanism. However, in the following example, we will consider the case of external forces on both the coupler and follower links and will utilize the principle of superposition.

Example Problem 9.2

Static-force analysis of a four-bar linkage is considered. The link lengths for the four-bar linkage of Figure 9.11A are given in the figure. In the position shown, coupler link 2 is subjected to force F_2 of magnitude 47 N, and follower link 3 is subjected to force F_3 of magnitude 30 N. Determine the shaft torque T_1 on input link 1 and the bearing loads for static equilibrium.

SOLUTION

As shown in Figure 9.11A, the solution of the stated problem can be obtained by superposition of the solutions of subproblems I and II. In subproblem I, force F_3 is neglected, and in subproblem II, force F_2 is neglected. This process facilitates the solution by dividing a more difficult problem into two simpler ones.

The analysis of subproblem I is shown in Figure 9.11B, with quantities designated by superscript I. Here, member 3 is a two-force member because force F_3 is neglected. The direction of forces $F_{23}{}^{I}$ and $F_{03}{}^{I}$ are as shown, and the forces are equal and opposite (note that the magnitude and sense of these forces are as yet unknown). This information allows the analysis of member 2, which is a three-force member with completely known force F_2, known direction for $F_{32}{}^{I}$, and, using the concurrency point, known direction for $F_{12}{}^{I}$. Scaling from the force polygon, the

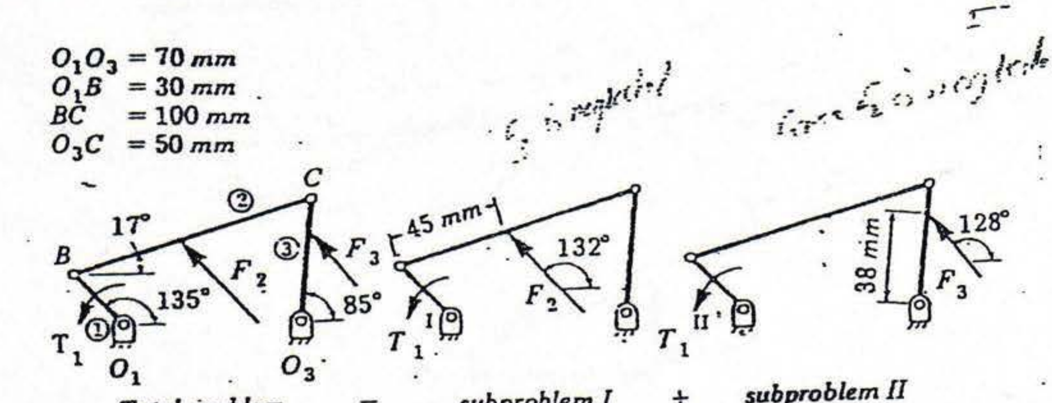


Figure 9.11(A) Graphical force analysis of a four-bar linkage, utilizing the principle of superposition.

following force magnitudes are determined (the force directions are shown in Figure 9.11B):

$$F_{32}{}^{\mathrm{I}} = F_{23}{}^{\mathrm{I}} = F_{03}{}^{\mathrm{I}} = 21 \text{ N}$$

$$F_{12}{}^{\mathrm{I}} = F_{21}{}^{\mathrm{I}} = 36 \text{ N}$$

Link 1 is subjected to two forces and couple $T_1{}^{\mathrm{I}}$, and for equilibrium,

$$F_{01}{}^{\mathrm{I}} = -F_{21}{}^{\mathrm{I}}$$

and

$$T_1{}^{\mathrm{I}} = F_{21}{}^{\mathrm{I}} h^{\mathrm{I}} = (36 \text{ N})(11 \text{ mm}) = 396 \text{ N} \cdot \text{mm cw}$$

The analysis of subproblem II is very similar and is shown in Figure 9.11C, where superscript II is used. In this case, link 2 is a two-force member and link 3 is a three-force member, and the following results are obtained:

$$F_{03}{}^{\mathrm{II}} = 29 \text{ N}$$

$$F_{23}{}^{\mathrm{II}} = F_{21}{}^{\mathrm{II}} = F_{01}{}^{\mathrm{II}} = 17 \text{ N}$$

and

$$T_1{}^{\mathrm{II}} = F_{21}{}^{\mathrm{II}} h^{\mathrm{II}} = (17 \text{ N})(26 \text{ mm}) = 442 \text{ N} \cdot \text{mm cw}$$

The superposition of the results of Figures 9.11B and 9.11C is shown in Figure 9.11D. The results must be added vectorially, as shown. By scaling from the

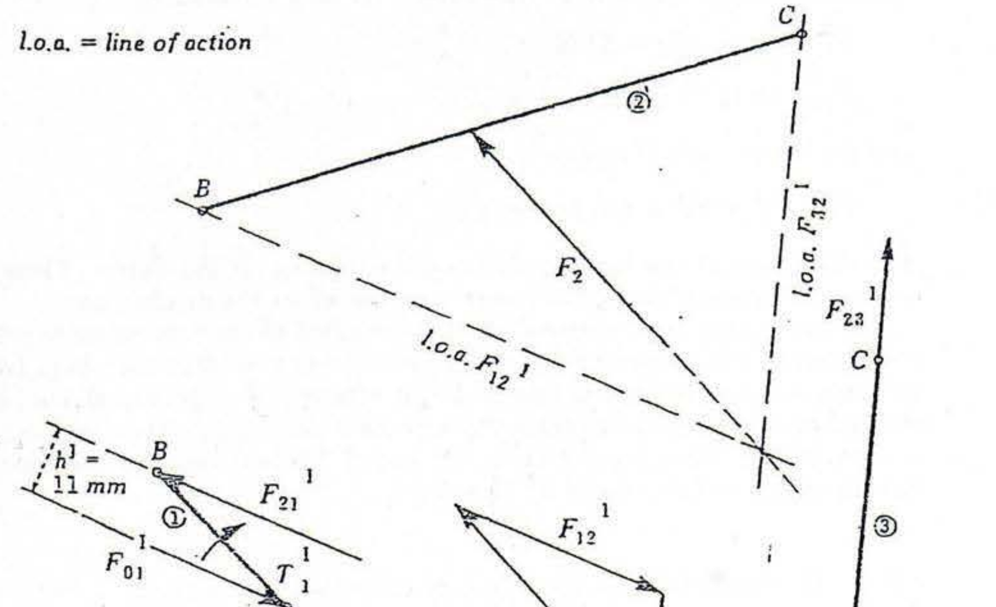


Figure 9.11(B) The solution of subproblem I.

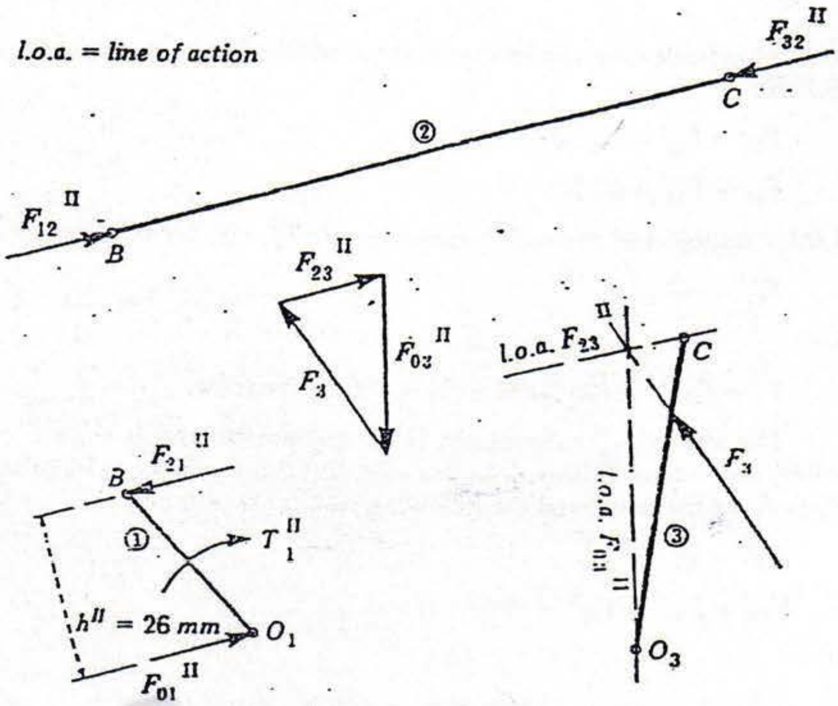


Figure 9.11(C) The solution of subproblem II.

free-body diagrams, the overall bearing force magnitudes are

$F_{01} = 50$ N $\quad F_{23} = 31$ N

$F_{12} = 50$ N $\quad F_{03} = 49$ N

and the net crankshaft torque is

$T_1 = T_1{}^{I} + T_1{}^{II} = 838$ N · mm cw

The directions of the bearing forces are as shown in the figure. These resultant quantities represent the actual forces experienced by the mechanism.

It can be seen from the analysis that the effect of the superposition principle, in this example, was to create subproblems containing two-force members, from which the separate analyses could begin. In an attempt of a graphical analysis of the original problem without superposition, there is not enough intuitive force information to analyze three-force members 2 and 3, because none of the bearing force directions can be determined by inspection.

9.3.6 Graphical Force Analysis of Complex Linkages

In this section, an example is presented involving the static-force analysis of a mechanical system that is somewhat more complex than the previous cases considered. This example will demonstrate that, although each force analysis problem has its own special characteristics, the solution procedure for a broad range of mechanisms is essentially unchanged, relying on the basic force analysis groundwork that has been developed.

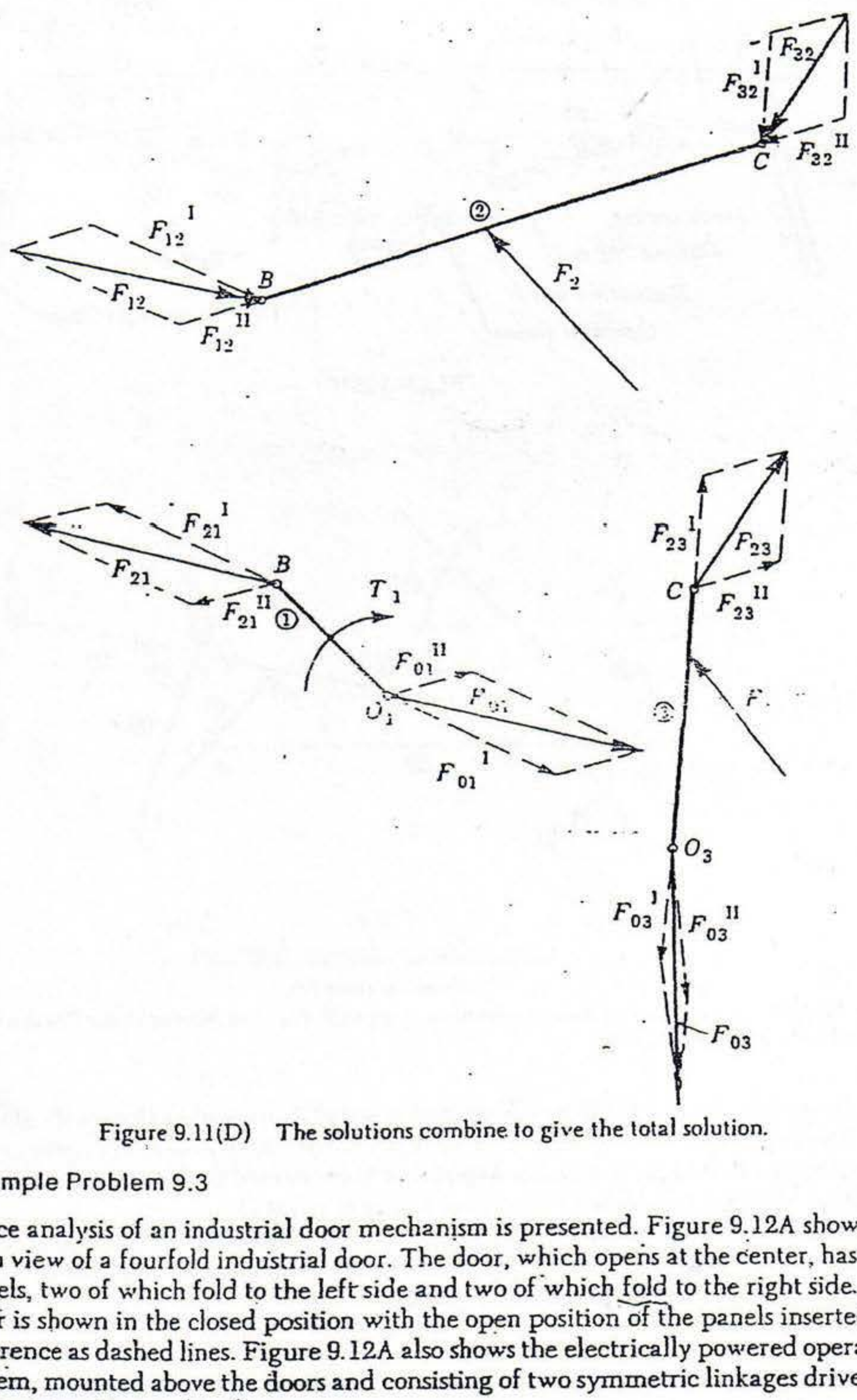
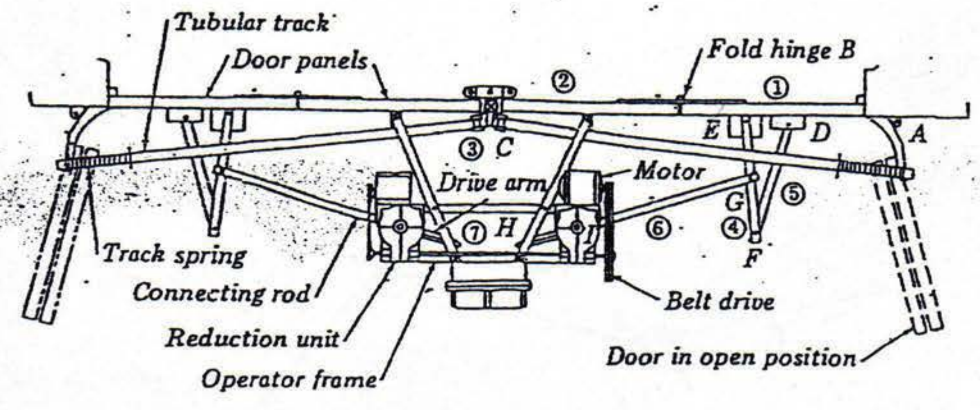


Figure 9.11(D) The solutions combine to give the total solution.

Example Problem 9.3

Force analysis of an industrial door mechanism is presented. Figure 9.12A shows the plan view of a fourfold industrial door. The door, which opens at the center, has four panels, two of which fold to the left side and two of which fold to the right side. The door is shown in the closed position with the open position of the panels inserted for reference as dashed lines. Figure 9.12A also shows the electrically powered operating system, mounted above the doors and consisting of two symmetric linkages driven by the same motor.

Figure 9.12B is a schematic drawing of the right half of the system, drawn to scale for an intermediate position between the open and closed door positions. Including the door frame and the two door panels as links, the mechanism is an eight-bar linkage. Member 0 is the frame and members 1 and 2 are the door panels



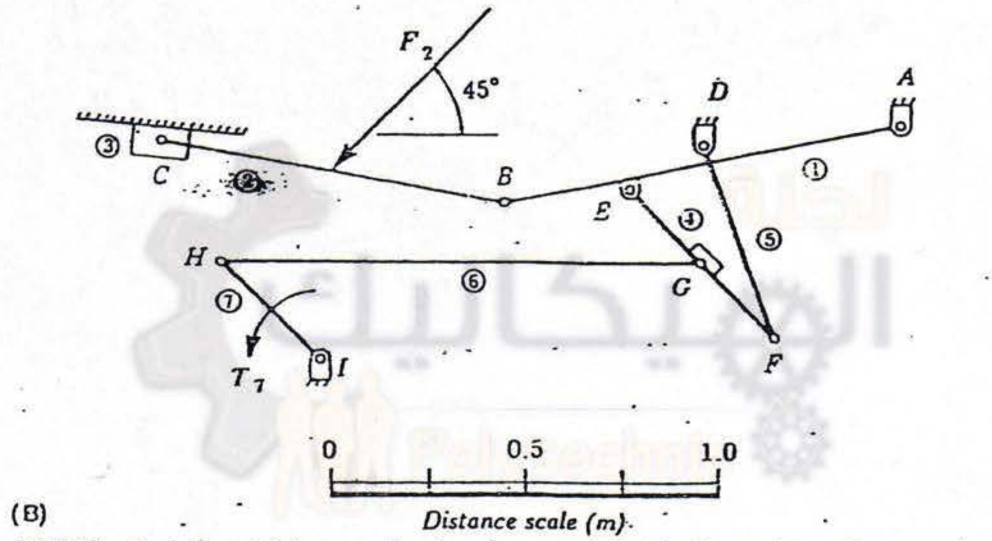


Figure 9.12(A) An industrial door mechanism. (SOURCE: Electric Power Door Company.) (B) Schematic linkage diagram.

hinged together at point B. Slider 3 is pinned to panel 2 and moves along a fixed track as the doors open and close. Power is transmitted to the door by means of drive arm 7, connecting rod 6, and links 4 and 5. Member 4 is connected to door panel 1 at point E, and member 5 is connected to the door frame at point D.

For the position of Figure 9.12B, determine the required shaft torque on drive arm 7 for static equilibrium against applied load F_2, which has a magnitude of 1000 N and acts on door panel 2 as shown in the figure.

SOLUTION

It is assumed that inertia forces and friction effects are negligible. A planar force analysis will be performed considering those forces that act in planes parallel to that shown in Figure 9.12B. Gravity loads, which act perpendicular to these planes, are not included in the analysis.

The graphical analysis, presented in Figure 9.12C, starts with slider 3, which is a

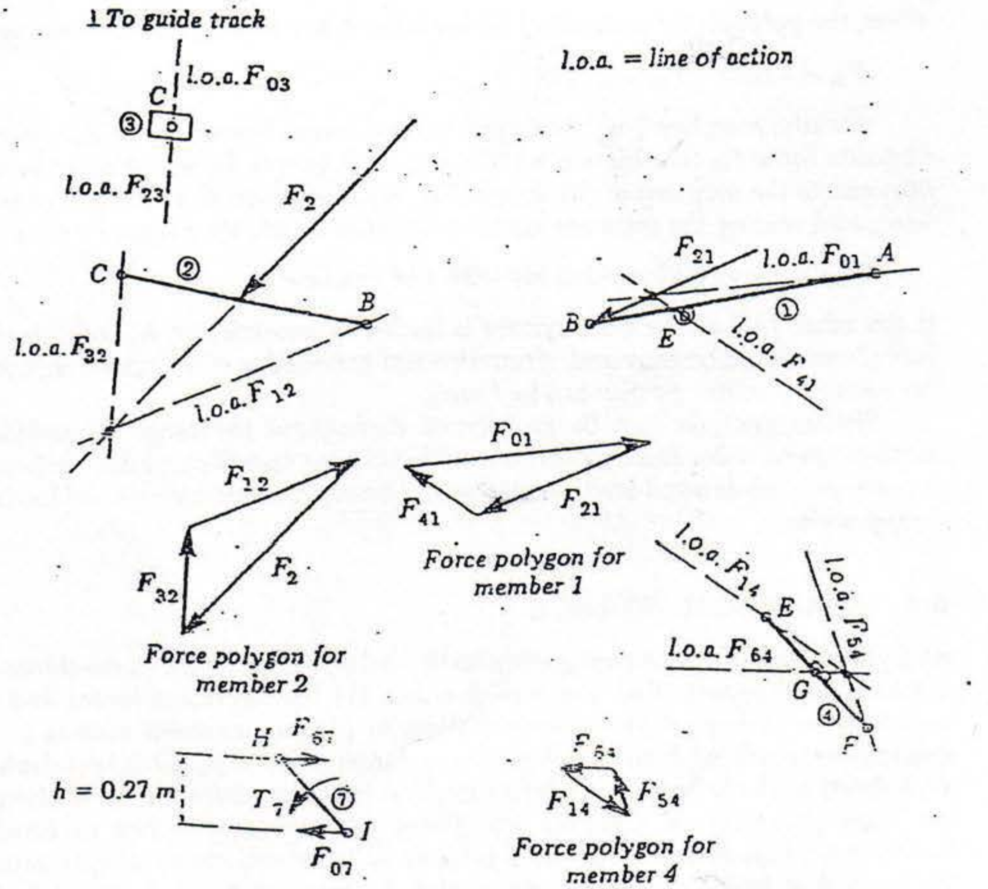


Figure 9.12(C) Graphical force analysis of an industrial door mechanism.

two-force member. Since friction is neglected, these forces must act perpendicular to the guide track, thus establishing the directions of forces F_{03} and F_{23}. Door panel 2 is a three-force member with known force F_2 and known direction of force F_{32}. From this information, the concurrency point can be found and the force polygon constructed, yielding the following force magnitudes:

$F_{32} = 420$ N $\quad F_{12} = 730$ N

Member 1 is also a three-force member, acted on by force F_{21}, which is now completely known, and the forces F_{01} and F_{41}, both of which have unknown direction and magnitude. In order that the analysis of this member be completed, the direction of either F_{01} or F_{41} must be determined.

The direction of F_{41} can be found by considering link 4, which is another three-force member, acted upon by force F_{14} from link 1 at point E, force F_{54} from link 5 at point F, and force F_{64} from link 6 at point G. Since links 5 and 6 are two-force members, the lines of action of forces F_{54} and F_{64} are along the respective links, and the intersection of these lines is the concurrency point for member 4 (see Figure 9.12C). This leads to the line of action for F_{14} and, in turn, the direction of F_{41}.

The force polygon can now be constructed for member 1, as shown in Figure 9.12C, yielding the following force magnitudes:

$F_{01} = 960$ N $\quad F_{41} = 350$ N

Next, the polygon is constructed for member 4 (see Figure 9.12C), from which

$F_{54} = 210\text{ N} \quad F_{64} = 220\text{ N}$

Finally, member 7 is acted upon by two forces, known force F_{67} and equal and opposite force F_{07} (see Figure 9.12C), and shaft torque T_7, which must be equal and opposite to the moment of the couple F_{67}, F_{07}. Therefore, the torque is counterclockwise, and, scaling the moment arm from Figure 9.12B, the magnitude is

$T_7 = hF_{67} = (0.27\text{ m})(220\text{ N}) = 59.4\text{ N}\cdot\text{m ccw}$

If the other half of the door system is loaded symmetrically, a total torque double that above would be required. From this and knowledge of the speed reduction unit, the necessary motor torque can be found.

Similar analyses can be performed throughout the range of motion of the mechanism in order to size components for proper operation under various loading conditions, such as wind loads, which would be represented by external loads on both door panels.

9.4 ANALYTICAL STATICS

Analytical methods for investigating static and dynamic forces in machines employ mathematical models that are solved either (1) for unknown forces and torques associated with known mechanism motion, or (2) for unknown motion of a given mechanism resulting from known driving forces or torques. This text deals almost exclusively with the former analysis category; however, there is a brief discussion of the latter category in Chapter 10. There are two approaches to formulating mathematical models; one approach is based on force and moment equilibrium, and the second is based on energy principles. Methods utilizing force and moment equilibrium equations parallel very closely the graphical method that has been presented. Both rely heavily on free-body diagrams, but the graphical force polygons are replaced in the analytical approach by equivalent vector equations. Energy methods utilize the principle of conservation of energy, one of the best-known examples being the method of virtual work.

The mathematical basis of the analytical approach lends itself well to computer implementation. Solutions can be obtained quickly and accurately for many positions of a mechanism, and the computer is particularly useful in design situations where many mechanism variations are to be considered. This facilitates design optimization, wherein those values of design parameters are determined such that selected performance criteria are optimized.

The designer may choose to write his own computer program for analysis or apply one of a number of general-purpose programs that are available. Examples of large programs that have been developed for various kinematic analysis, static-force analysis, and dynamic-force analysis tasks associated with planar and/or spatial machinery are Automatic Dynamic Analysis of Mechanical Systems (ADAMS),[1] Dynamic Response of Articulated Machinery (DRAM),[2] and Integrated Mechanisms Program (IMP).[3] These computer codes are very general and, therefore, are applicable to a broad range of mechanical systems.

[1]Marketed by Mechanical Dynamics Inc., Ann Arbor, Michigan.
[2]Marketed by Mechanical Dynamics Inc., Ann Arbor, Michigan.
[3]Marketed by Structural Dynamics Research Corporation, Cincinnati, Ohio.